

Exercice 21

$$\cos(\mathbf{p} - \mathbf{a}) = \cos(\mathbf{p} + \mathbf{a}) = -\cos(\mathbf{a}) \text{ l'égalité est donc vraie}$$

$$\sin\left(\frac{\mathbf{p}}{2} - \mathbf{a}\right) = \cos(-\mathbf{a}) = \cos(\mathbf{a}) \text{ l'égalité est donc vraie}$$

$$\cos(\mathbf{p} + \mathbf{a}) = \sin\left(\frac{\mathbf{p}}{2} + \mathbf{a}\right); \cos(\mathbf{p} + \mathbf{a}) = -\cos(\mathbf{a}) \text{ et } \sin\left(\frac{\mathbf{p}}{2} + \mathbf{a}\right) = \cos(\mathbf{a})$$

l'égalité est donc fausse

$$\tan(\mathbf{p} - \mathbf{a}) = \cos(\mathbf{a}); \tan(\mathbf{p} - \mathbf{a}) = -\tan(\mathbf{a}); \text{l'égalité est donc fausse}$$

$$\cos\frac{3\mathbf{p}}{4} = -\frac{\sqrt{2}}{2}; \text{ direct(lecturesur cercle); vraie}$$

$$\sin\frac{2\mathbf{p}}{3} + \tan\frac{3\mathbf{p}}{4} = -\frac{1}{2}; \sin\frac{2\mathbf{p}}{3} + \tan\frac{3\mathbf{p}}{4} = \frac{\sqrt{3}}{2} - 1 \neq -\frac{1}{2}$$

Exercice 22 on prendra pour tout l'exercice $k \in \mathbb{Z}$

$$1) \cos\left(2x + \frac{\mathbf{p}}{3}\right) = -\frac{\sqrt{3}}{2} = \cos\left(\frac{5\mathbf{p}}{6}\right) \Leftrightarrow \begin{cases} 2x + \frac{\mathbf{p}}{3} = \frac{5\mathbf{p}}{6} + 2k\mathbf{p} \\ 2x + \frac{\mathbf{p}}{3} = -\frac{5\mathbf{p}}{6} + 2k\mathbf{p} \end{cases} \Leftrightarrow \begin{cases} x = \frac{\mathbf{p}}{4} + k\mathbf{p} \\ x = -\frac{7\mathbf{p}}{12} + k\mathbf{p} \end{cases}$$

$$2) \cos(2x) = \cos(\mathbf{p} + 3x) \Leftrightarrow \begin{cases} 2x = \mathbf{p} + 3x + 2k\mathbf{p} \\ 2x = -\mathbf{p} - 3x + 2k\mathbf{p} \end{cases} \Leftrightarrow \begin{cases} x = \mathbf{p} + 2k\mathbf{p} \\ x = -\frac{\mathbf{p}}{5} + \frac{2k\mathbf{p}}{5} \end{cases}$$

$$3) \sin 3x = \cos(x + \mathbf{p}) = \cos\left(\frac{\mathbf{p}}{2} - 3x\right) \Leftrightarrow \begin{cases} x = -\frac{\mathbf{p}}{8} + \frac{k\mathbf{p}}{2} \\ x = \frac{3\mathbf{p}}{4} + k\mathbf{p} \end{cases}$$

$$4) \sin^2 x = \frac{1}{4} \Leftrightarrow \sin x = \frac{1}{2} \text{ ou } \sin x = -\frac{1}{2} \Leftrightarrow \begin{cases} x = \frac{\mathbf{p}}{6} + k\mathbf{p} \\ x = -\frac{\mathbf{p}}{6} + k\mathbf{p} \end{cases}$$

Exercice 23 :

$$\text{On note } B = \cos\left(-\frac{11\pi}{4}\right) + \cos\left(\frac{7\pi}{4}\right) + \cos\left(\frac{23\pi}{4}\right) \text{ et } C = \sin\left(-\frac{7\pi}{6}\right) + \sin\left(\frac{5\pi}{6}\right) + \sin\left(\frac{17\pi}{6}\right)$$

$$B = \cos\left(-\frac{11\pi}{4}\right) + \cos\left(\frac{7\pi}{4}\right) + \cos\left(\frac{23\pi}{4}\right) = \dots = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$$

$$C = \sin\left(-\frac{7\pi}{6}\right) + \sin\left(\frac{5\pi}{6}\right) + \sin\left(\frac{17\pi}{6}\right) = \dots = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$$

$$\text{Donc } A = \frac{B}{C} = \frac{\sqrt{2}}{3}$$